APPLICATION OF RANK CONTROLLED DIFFERENTIAL QUADRATURE METHOD FOR SOLVING AN INFINITE STEEL PLATE COOLING PROBLEM

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1. Introduction

The heat transfer phenomenon is described by Fourier – Kirchhoff (F-K) Partial Differential Equation (PDE) with geometrical, physical, boundary and initial conditions [1, 2] . In most cases it is impossible to find the analytical solution of this system, as the solving requires complex transformations which often leads to non-elementary functions. However in some cases it is possible to introduce mathematical model that describe simple heat transfer problem which can be solved exactly [3]. An example of such problem is cooling of infinite plate under assumption of constant thermo-physical parameters [4]. Knowledge of exact solution allows one to analysis of numerical solution quality.

A Rank Controlled Differential Quadrature (RCDQ) method is numerical method for solving PDEs [5]. This method come into being as the adaptation of Differential Quadrature method [6] for solving heat transfer problems on dense, equidistant grids. The aim of this work is to apply the Rank Controlled Differential Quadrature method for solving an infinite steel plate cooling problem and confront numerical solution with the exact one.

2. Mathematical and numerical model

The temperature change in cooled infinite steel plate is a problem that can be described with 1D F-K equation, after this simplification it is symmetrical problem. In order to reduce the computation effort only half of the domain is taken under consideration. Mathematical model for this problem is:

$$\begin{cases} \dfrac{\partial T(x,\tau)}{\partial \tau} = \dfrac{\lambda}{c_p \rho} \dfrac{\partial^2 T(x,\tau)}{\partial x^2}, & \tau \geq 0, \quad x \in [0,\ L], \\[2mm] \dfrac{\partial T(0,\tau)}{\partial x} = 0, \\[2mm] -\lambda \dfrac{\partial T(L,\tau)}{\partial x} = \hat{\alpha}\big(T(L,\tau) - T_{amb}\big), \\[2mm] T(x,0) = T_{init}, & x \in [0,\ L], \end{cases} \quad (1)$$

where: λ [W m^{-1} K^{-1}] denotes thermal conductivity, c_p [J kg^{-1} K^{-1}] specific heat, ρ [kg m^{-3}] density, $\hat{\alpha}$ [W m^{-2} K^{-1}] heat transfer coefficient, $T(x,\tau)$ [K] function of temperature values,

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x [m] spatial variable, τ [s] time variable, $2L$ [m] thickness of plate, T_{init}, T_{amb} [K] initial and ambient temperature respectively.

Given model under assumption of constant thermo-physical parameters can be solved exactly [3, 4].

3. Results

Basing on algorithm created with RCDQ method computer program was prepared. Computations were made for $\lambda = 35$ W/(m K), $c_p = 690$ J/(kg K), $\rho = 7500$ kg/m^3 [1], $2L = 0.2$ m, $\hat{\alpha} = 250$ W/(m^2 K), $T_{init} = 450$ °C, $T_{amb} = 23$ °C. In numerical model equidistant grid of $N = 300$ nodes in interval [0, 0.1] was introduced, time interval was chosen to be equal $\Delta\tau = 0.5\ 10^{-7}$ s. Chosen results are shown in figures Fig. 1a,b.
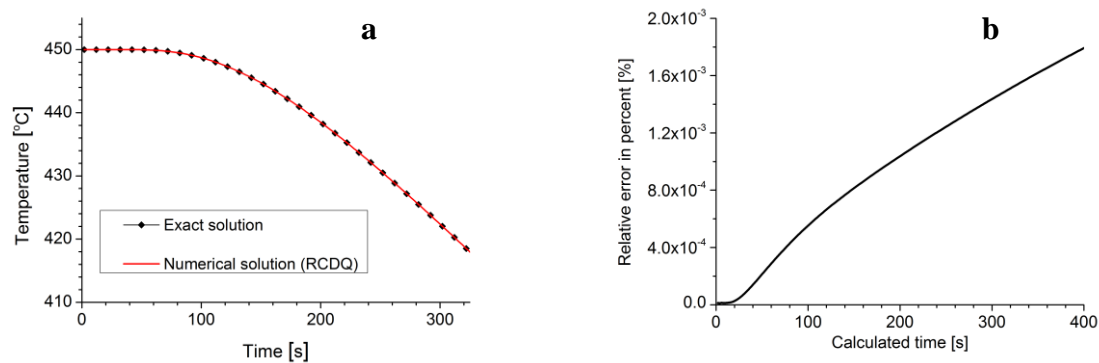
Fig. 1a. Comparison of numerical and exact solution of infinite plate cooling problem.
Fig. 1b. Relative percentage error increase during numerical simulation of infinite steel plate cooling. Error values are shown in calculated moments of time

4. Conclusions

Knowledge of exact solution of problems described with PDEs allows to test quality of numerical solution. Which is especially important for the newly developed methods.

The RCDQ method can be used to solving the heat transfer problems. The numerical solution given by the RCDQ method based simulation is of high accuracy.

Numerical approximation error increase during simulation. This may be caused by summing errors that appear at each time step.

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