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| agh_znk_wbr_rgb_150ppi | FACULTY OF FOUNDRY ENGINEERINGINTERNATIONAL WORKSHOP ***90 Years of Educating Foundry Engineers by the***  ***AGH University of Science and Technology in Krakow*** connected withXXXVI SCIENTIFIC CONFERENCE FOUNDRYMAN'S DAY 2012Krakow, 22 – 23 Nov. 2012 |  |
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THE RANK CONTROLLED DIFFERENTIAL QUADRATURE - A NUMERICAL METHOD IMPROVED FOR FOUNDRY ENGINEERING

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**Keywords**: Numerical methods, Rank Controlled Differential Quadrature, Differential Quadrature, mathematical model

**Abstract**

The Differential Quadrature [1] method is high rank numerical method. It can be applicable in building numerical models for Partial Differential Equations (PDE) [2]. However for dense grids, especially for the equally-distanced nodes, method loses its accuracy[3].

The aim of the project was to construct such modification of DQ that can improve its accuracy for problems that appears in foundry engineering. Presented in the work Rank Controlled Differential Quadrature (RCDQ) [4-6] is searched adjustment of DQ that make it applicable it in the foundry processes numerical analysis.

During numerical modelling with RCDQ the critical is to define the function that linking nodes with intended approximation rank. With this function computational domain decomposes into a sets of nodes. Sets are numbered and the index connects them with a point for value in which numerical scheme is applied[4-6]. In such limited numerical domains classical DQ [7,8] scheme is adopted.

Analyse of the numerous numerical experiments proves that the solution quality is improved with RCDQ. Experiments focused on three main subjects:

- accuracy of derivative approximation;

Introduced method improves accuracy of numerical solution. Problems with accuracy near to domain borders that can be observed for DQ are no longer crucial for RCDQ. Distribution of an error in domain is similar (in shape) as in the case of Finite Differences (FD) method, but some ranks of order lower (in value).

- comparison of numerical and exact solution for mathematical model of simplified heat transfer problem;

The RCDQ method was also applied to solve the problem of infinite steel plate cooling from 450 °C that was cooled in ambient temperature of 23 °C with 3rd type of boundary condition. The numerical results shows very high agreement to exact ones.

- application of RCDQ in magnesium based solidification problem.

The RCDQ method was applied to solve real life problem: solidification of magnesium based alloy. After several simplifications mathematical model was created. This model was solved with analyzed numerical method. The results compared with experimental data show that RCDQ may be successively used to solving foundry engineering problems including the solidification phenomenon.

**Conclusions**

The assumed goal of the work was fulfilled:

- modification of DQ method leads to new numerical method RCDQ that can be applied in foundry technology;

- RCDQ method has high accuracy even for dense grids in comparison to FD and DQ methods;

- effective macro-model of solidification was introduced that with RCDQ builds numerical model for alloy solidification analysis.

**Acknowledgements**

The authors acknowledge financial support from the Polish Ministry of Science and Higher Education under Dean's Grant AGH No. 15.11.170.451. A special thanks goes to Professor W. K. Krajewski, PhD. J. Lelito and M. Sc. B. Gracz for significant effort during this manuscript preparation.

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